

Final Exam , MTH 101, Spring 2015

Ayman Badawi

Part I

(i) (2 points) In a group of 42 students, 22 take history, 17 take biology and 8 take both history and biology. How many students take biology but not history?

(ii) (2 points) How many 8-letter words are possible if the letters are chosen from the first 6 letters of the alphabet and the first letter cannot be a 'B" and adjacent letters cannot be the same?

(iii) (2 points) A software company employs 9 sales representatives and 8 technical representatives. How many ways can the company select 6 of these employees to send to a computer convention if at least 5 technical representatives must attend the convention?

(iv) (2 points) How many ways can a committee of 6 be selected from a club with 10 members if Ali and Bill are both on the committee?

(v) (2 points) Two 6-sided dice are rolled. What is the probability the sum of the two numbers on the dice will be 4, P(sum is 4)?

(vi) A study conducted at a large state university involved 400 graduates of business related programs. The grades they received in a required Mathematics course were determined along with their majors. The results are given in the following table:

Program	Grade A	B	C or lower	Total
Accounting	30	40	30	100
Management	0	20	80	100
Other	30	60	110	200
Total	60	120	220	400

a. (2 points) How many students are accounting majors or get C or lower in the Mathematics course?

b. (2 points) What is the probability that a student is an accounting major and gets at least a B?

(vii) (2 points) A catering service offers 8 appetizers, 10 main courses, and 6 desserts. A banquet committee is to select 3 appetizers, 4 main courses, and 2 desserts. In how many ways can this be done?

(viii) (2 points) A group of friends is planning an evening out. They have a choice of 4 restaurants for dinner, 6 movies following dinner, and 4 coffee establishments for after the movie. They choose one of each randomly. What is the probability that you will correctly predict the three choices just by guessing?

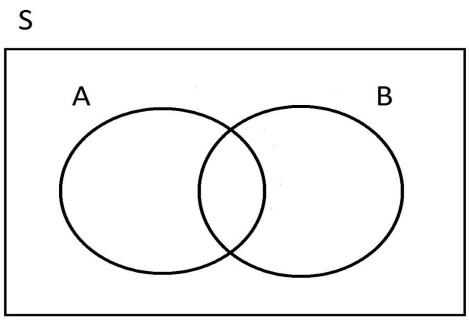
(ix) A store chain has 15 stores in Sharjah, 10 in Dubai, and 5 in Abu Dhabi. It is planing to randomly close 10 of these stores.

a. (2 points) What is the probability that 2 will be closed in Sharjah, 7 in Dubai and 1 in Abu Dhabi?

b. (2 points) What is the probability that none of Sharjah stores will be closed?

(x) (2 points) A Shipment of 60 computers contains 10 which are defective. 5 computers are selected at random. The shipment will be rejected if at least 1 computer is defective. What is the probability that the shipment will be rejected?

(xi) (2 points) Assume that $P(A) = 0.30$, $P(B) = 0.55$, and $P(A \cup B) = 0.70$. Find the probability of each of the four disjoint subsets in the following Venn diagram. In other words, find the following four probabilities: $P(A \cap B')$, $P(A \cap B)$, $P(A' \cap B)$, $P(A' \cap B')$



Part II

(i) (9 pts) The following **tableau** was obtained while solving a **maximization** problem using the **simplex method**. Continue solving. Find the maximum value if possible.

$$\left[\begin{array}{cccccc|c} 1 & 1 & 1 & 1 & 0 & 0 & 40 \\ 2 & 1 & -1 & 0 & 1 & 0 & 10 \\ \hline -1 & -3 & -1 & 0 & 0 & 1 & 0 \end{array}\right]$$

(ii) (6 pts) You want to invest \$6000 for **15** years. You have the choice between the following three different investments

Bank A offers 8% compounded semi annually.

Bank B offers 7.9% compounded quarterly.

Bank C offers 8.2% of simple interest.

Which bank should you choose. Justify your answer.

(iii) (7 points) A couple wishes to set up a retirement account by depositing $\$20,000$ at the end of every year over the next 30 years. Then, the couple will not make any deposits or withdrawals during the following 10 years. Assuming that the interest rate is 9% compounded annually during the entire 40 years, find the final bank balance of the account.

(iv) (8 pts) You make equal deposits at the end of every month into an account that pays an interest rate of 9% compounded monthly. Assume that, after 4 years, the amount of money in the bank account is $\$48,000$.

a. What are the monthly deposits? (Round your answer to two decimal places)

b. How much interest will you earn during the $5th$ year [from year 4 to year 5]? (Assume that the initial balance of the account was zero.)

(v) (12 pts) Let x be the number of a particular item in hundreds. Assume that the price-demand per item and the total cost functions are given by

$$p(x) = 40 - 0.5x,$$
$$C(x) = 250 + 10x.$$

where $0 \leq x \leq 80$.

a. What is the profit function?

b. How many items are needed in order to get the maximum profit?

c. What should be the selling price per item in order to get the maximum profit?

d. Find the Break Even Points.

e. For what values of x does a loss occur? A profit?

(vi) (4 pts) Find all solution(s), if any, to the following system of linear equations.

$$\begin{aligned} x + z &= 4 \\ 2x + y + 2z &= 10 \\ 3x + 4z &= 13 \end{aligned}$$

(vii) (4 pts) A trucking company needs to purchase three types of trucks: small, medium and large. Let $x =$ the number of small trucks purchased, $y =$ the number of medium trucks purchased, and $z =$ the number of large trucks purchased. The company determined that x, y and z satisfy the following system of linear equations:

$$\begin{aligned} x - \frac{1}{2}z &= -6 \\ y + 3z &= 60 \end{aligned}$$

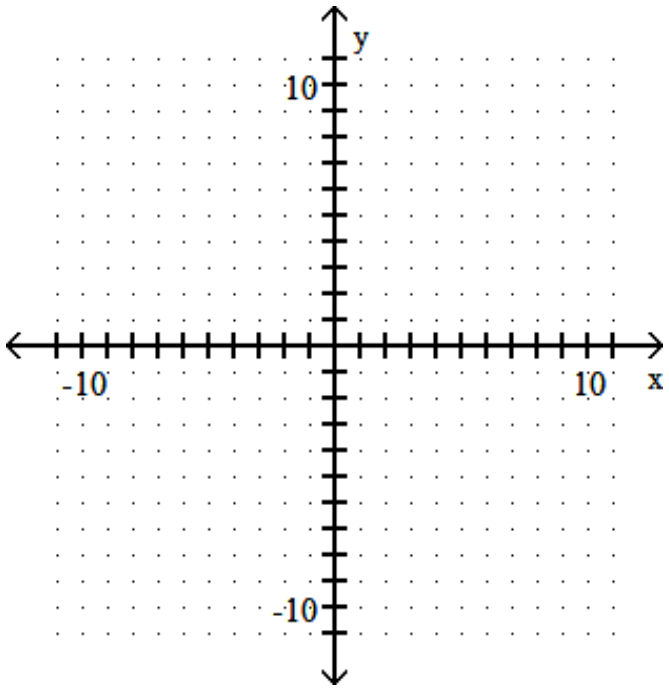
Find all solution(s), if any.

(viii) (4 pts) Solve the following linear programming problem using the geometric approach. Maximize and Minimize $P = 7x + 14y$ Subject to:

$$2x + y \geq 10$$

$$x + 4y \geq 8$$

$$x, y \geq 0$$



(ix) (5 pts) Consider the following linear programming problem:

$$\begin{aligned} &\text{Minimize } C = x_1 + 4x_2 \\ &\text{Subject to:} \\ &x_1 - 4x_2 \geq -9 \\ &-3x_1 + 2x_2 \geq 5 \\ &x_1, x_2 \geq 0 \end{aligned}$$

a. STATE the **dual problem** (but do not solve)

b. Construct the **initial tableau** and locate the pivot column and the pivot value (again, do not solve it)

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